Name of Course : **CBCS (LOCF) B.Sc. (Hons) Mathematics**

Unique Paper Code : **32357507**

Name of Paper **DSE-2: Probability Theory and Statistics**

Semester : **V**

Duration : **3 hours**

Maximum Marks : **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Suppose that the cumulative distribution function of the random variable X is given by

$$F(x) = 1 - e^{-x^2}, x > 0.$$

Evaluate P(X>2), E(X) and Var(X). Find the 25^{th} percentile(pth percentile is a value ξp such that P(X < ξp) ≤ p and P(X ≤ ξp) ≥ p) , the mode and the median of this distribution.

2. Let C be the set of points interior to or on the boundary of a square with side of length 1. Moreover, say that the square is in the first quadrant with one vertex at the point (0, 0) and an opposite vertex at the point (1, 1). Let P(A) be the probability of region A contained in C. If A={(x, y): 0<x<y<1}, compute P(A), and what will be P(A) if A={(x, y): 0<x=y<1}. Suppose, two points are independently chosen at random in the interval (-1, 1). Obtain the probability that the three parts into which the interval is divided can form the sides of a triangle.

3. State the memory-less property of the exponential distribution. Let the time (in hours) required to repair a smart mobile is exponentially distributed with mean 3. What is the probability that the repair time exceeds 3 hours? Also, find the probability that a repair takes at least 5 hours given that its duration exceeds 4 hours?

4. Let

$$f(x, y) = 24xy,\ 0<x<1,\ 0<y<1,\ 0< x + y <1, \text{ and } =0, \text{ otherwise.}$$

Find the moment generating function of X and Y, and hence, find whether X and Y are independent? Further obtain the coefficient of correlation between X and Y.

5. Let

$$f(x, y) = 10xy^2,\ 0 < x < y<1, \text{ and } =0 \text{ elsewhere, be the joint pdf of X and Y.}$$

Find the conditional mean and variance of X, given Y=y, 0 < y <1. Hence find the distribution of Z=E(X|Y) and determine E(Z) and Var(Z) and compare these to E(X) and Var(X), respectively.

6. (i) State the Chebyshev's Theorem (or Inequality). Let the number of customer's visiting a bike showroom is a random variable with mean 12 and standard deviation 2. With what probability can we assert that there will be more than 6 but fewer than 18 customers visiting the showroom?

(ii) Let $\{X_i\}$, i=1, 2, … be a sequence of i.i.d. Poisson variables with $E[X_i]=1.5$. Find $P(160 < Y < 200)$, where $Y = X_1 + X_2 + \ldots + X_{100}$